The 6th Competition of Trajectory Optimization of China (CTOC) Problem Description

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Introduction

The 6th Competition of Trajectory Optimization of China (CTOC) (the former Competition of Deep Space Trajectory Optimization of China) is held by the Chinese Society of Theoretical and Applied Mechanics and State Key Laboratory Astronautic Dynamics, affiliated to Xi'an Satellite Control Centre. The 1st (2009) CTOC was held by the Chinese Society of Theoretical and Applied Mechanics and School of Aerospace Engineering of Tsinghua University. As usual, the champion team of this competition will get the opportunity to hold the 7th CTOC (independently or in form of cooperation). The deadline of result submission of this competition is 24:00 CST, 31st August 2014.

1. Problem Description

The task background is asteroid sample and return mission. The probe is to depart from a 200km circular Low Earth Orbit (LEO) at sometime between 1st January 2021(MJD59215) and 31st December 2030(MJD62867), and return to the earth after rendezvous with the asteroids (choose from the asteroids data file) with sample taken back from it. The probe's position and velocity with allowable error at the moment of rendezvous should be the same as that of the asteroid and the stay time should be at least 30 days. At the moment of reentry to the Earth, the probe is required to be 6578km away from the Earth center with the velocity no more than 11.0 km/s in the Earth Center Inertial Frame (ECI). The departure position from LEO is optional and the inclination of LEO is between 20 degree and 90 degree. The initial mass of the probe is 2000kg, which includes 1500kg of fuel. The trajectory of the probe is influenced by the gravitation of the earth, the moon and the sun; the gravitation of asteroid and other planets (except the Earth) is not in consideration. The total flight time should be no more than 10 years.

There are two candidate types of thruster of the probe could be chosen. In the final submission of result, each team is allowed to choose one type of the thruster. Type 1: Electrical thruster with small thrust magnitude. The specific impulse is 3000s and maximum thrust is 10N. The value and direction of thrust could be decided by optimization. Type 2: Chemical thruster with large thrust magnitude. The specific impulse is 400s and each maneuver could be simplified into instantaneous impulse. The value and direction of impulse could be decided by optimization and there is no limit to the time and number of thrusts.

2. Performance Index and Evaluation Criterion

The design result is required to be submitted before deadline without violation of any constraints. The result that violates the constraints seriously or is submitted after deadline is not acceptable. Whether the design has serious violation of constraints or not is decided by the organizer of competition by checking the data of the result.

Performance index is to maximize the mass of sample from the asteroids, which is

$$\text{Maximize: } J=m_{\text{Asteroid}} \qquad (1)$$

The mass of the sample from asteroids is counted in integer, and the unit is kg. If the performance index is the same, then the one with more remaining mass after reentry to the Earth is considered to be a better result.

3. Summary and Description of Constraints

Dynamic model is set up and described in the Earth Center Inertial Frame (ECI) (refer to Appendix A). The orbits of the Sun and the Moon relative to the Earth and the orbits of the asteroid relative to the sun are considered as two-body problem. Please refer to Orbitelements_Sun&Moon.txt and Orbitelements_Asteroids.txt to see the document of the orbits elements. The orbits elements of the sun and the moon are provided in ECI, from which the position and velocity of the sun and the moon in ECI could be obtained. The orbits elements of asteroid are given in the J2000 Heliocentric Ecliptic Inertial Reference Frame (HEIRF). And the position and velocity in HEIRF will be converted to which in ECI for calculation. Constraints include:

1) The restriction of the departure time from the earth:
   Time of departure from the earth (t_0 is between 1st January 2021, 00:00 ~ 31st December 2030, 24:00.
2) The restriction of the probe's initial state:
   At the time of departure from LEO t_0, the probe is at a circular Low Earth Orbit with altitude of 200km (radius of 6578km). The departure position can be chosen without any restriction, but the inclination should be between 20 degree and 90 degree.

$$\|\boldsymbol{r}_{sc}\| = 6578\ \text{km},\quad \|\boldsymbol{v}_{sc}\| = \sqrt{\mu_E / \|\boldsymbol{r}_{sc}\|} \tag{2}$$

$$\boldsymbol{r}_{sc} \cdot \boldsymbol{v}_{sc} = 0 \tag{3}$$

$$\pi/9 \le \arccos\left[\boldsymbol{k} \cdot (\boldsymbol{r}_{sc} \times \boldsymbol{v}_{sc}) / \|\boldsymbol{r}_{sc} \times \boldsymbol{v}_{sc}\|\right] \le \pi/2,\ \boldsymbol{k} = [0\quad 0\quad 1]^T \tag{4}$$

Where, μ_E represents the gravitation constant of the Earth (refer to Appendix B).

3) The restriction of the probe's position relative to the moon:

$$\|\boldsymbol{r}_{sc} - \boldsymbol{r}_M\| \ge 1838\ \text{km} \tag{5}$$

In which, $\boldsymbol{r}_M$ represents the moon position vector in ECI.

4) The restriction of the distance between the probe and the Earth:

$$\|\boldsymbol{r}_{sc}\| \ge 6578\ \text{km} \tag{6}$$

5) Ignoring the size of the asteroid, rendezvous with the asteroid requires:

$$\|\boldsymbol{r}_{sc} - \boldsymbol{r}_A\| \le 100\ \text{km},\quad \|\boldsymbol{v}_{sc} - \boldsymbol{v}_A\| \le 1\ \text{m/s} \tag{7}$$

Where $\boldsymbol{r}_A = \boldsymbol{r}_S + [\mathbf{M}]\boldsymbol{r}_{A\text{-}S}$, $\boldsymbol{r}_A$ and $\boldsymbol{r}_S$ represent the position vectors of the asteroid and the sun in ECI respectively, $\boldsymbol{r}_{A\text{-}S}$ represents the position vector of the asteroid in HEIRF, $[\mathbf{M}]$ represents the transitional matrix from HEIRF to ECI. For convenience, the coordinate vectors of the ECI are with the same directions to the HEIRF, so the $[\mathbf{M}]$ here is unit matrix.

6) The stay time on asteroid should be at least 30 days

$$t_{\text{departure}} - t_{\text{arrival}} \ge 30\ \text{days} \tag{8}$$

At the moment of departure from the asteroid, the increment of mass of the probe is m_{Asteroid}. Here m_{Asteroid} represent the mass of sample.

$$m_{sc}\left(t_{departure}\right)-m_{sc}\left(t_{arrival}\right)=m_{Asteroid} \tag{9}$$

7) The restriction of the end state of the probe

   At the moment of reentry to the Earth (t_f),

$$\left\|\boldsymbol{r}_{sc}\right\|=6578 \text{ km}, \quad \left\|\boldsymbol{v}_{sc}\right\|\le 11 \text{ km/s} \tag{10}$$

   The final mass (exclude the mass of sample) should not be less than 500kg, i.e.

$$m_{sc}\left(t_f\right)-m_{Asteroid}\ge 500 \text{ kg} \tag{11}$$

8) The total flight time should be no more than 10 years, i.e.

$$t_f-t_0\le 10 \text{ years} \tag{12}$$

9) Refer to Appendix A to see the dynamic model of probe.

Note: in the formulae above, $\boldsymbol{r}$ is position vector, $\boldsymbol{v}$ is velocity vector, and both of them are described in ECI. Subscript *sc* represents the probe, *S* represents the sun, *M* represents the Moon and operator ||▪|| represents the module of the vector.

4. Requirement of Result Submission

1) Submit a technical document in form of word or PDF. Technical document includes the brief introduction of the chosen of thruster mode (electrical thruster or chemical thruster), the method used and the design result. The design result should include at least these following parameters: the moment of departure from the Earth (MJD), the information of rendezvous with the asteroid (moment (MJD), position (km) and velocity (km/s)), the moment at which the distance between the probe and the moon is the smallest (MJD) and the corresponding distance (km). Meanwhile, designers should also provide the figures of the orbit.
2) Submit the orbital data in form of text document (sc_orbit.txt). Divide the orbit of probe into several orbit sections. Each orbit section represents the orbit between two arbitrary events (e.g., the event could be defined as departure from the Earth, and rendezvous with the asteroid). For each orbit section, please label serial

number and relative events. From column 1st to column 11th, please provide in order: time (MJD), position (three dimensions x, y, z; unit: km), velocity (three dimensions x, y, z; unit: km/s), mass of probe (unit: kg) and thrust vector (three dimensions x, y, z; unit: N, for electrical thruster) or instantaneous velocity impulse (three dimensions x, y, z; unit: km/s, for chemical thruster).

sc_orbit.txt data format:
coordinate system: Earth Center Inertial Frame (ECI)
thrust mode: electrical thrust/chemical thrust
1st section
description: Earth - Asteroid
#
slide section
59215 x y z vx vy vz m T_x T_y T_z (or Δv_x Δv_y Δv_z)
59216 x y z vx vy vz m T_x T_y T_z (or Δv_x Δv_y Δv_z)
…
thrust section
…
59315 x y z vx vy vz m T_x T_y T_z (or Δv_x Δv_y Δv_z)
59316 x y z vx vy vz m T_x T_y T_z (or Δv_x Δv_y Δv_z)
slide section
59316 x y z vx vy vz m T_x T_y T_z (or Δv_x Δv_y Δv_z)
…
59616 x y z vx vy vz m T_x T_y T_z (or Δv_x Δv_y Δv_z)
2nd section
description: asteroid—earth
slide section
59646 x y z vx vy vz m T_x T_y T_z (or Δv_x Δv_y Δv_z)
59647 x y z vx vy vz m T_x T_y T_z (or Δv_x Δv_y Δv_z)
…
thrust section
59656 x y z vx vy vz m T_x T_y T_z (or Δv_x Δv_y Δv_z)
…
59856 x y z vx vy vz m T_x T_y T_z (or Δv_x Δv_y Δv_z)
59857 x y z vx vy vz m T_x T_y T_z (or Δv_x Δv_y Δv_z)
slide section

59857 x y z vx vy vz m T_x T_y T_z (or Δv_x Δv_y Δv_z)

…

60168 x y z vx vy vz m T_x T_y T_z (or Δv_x Δv_y Δv_z)

For convenience of verification, the time step of data should be at most 1 day (shorter step is recommended when the spacecraft is near a gravity body). Please ensure that you have kept enough significant digits; double precision is highly recommended. The precision of numerical integration method should be not less than that of the fourth order Ronge-Kutta method.

Note: Please submit 1 technical document and 1 data document: sc_orbit.txt.

Appendix A. Dynamic Model of the Probe

The kinetic equations of the spacecraft are provided in the Earth Central Inertial Frame (ECI), considering the influence of the gravities of the Sun, the Earth and the Moon. And the equations are shown below:

1) Electrical thrust mode

$$\ddot{\boldsymbol{r}} = -\frac{\mu_{\mathrm{E}}}{r^3}\boldsymbol{r} - \mu_{\mathrm{M}}\left(\frac{\boldsymbol{r}_{\mathrm{M}}}{r_{\mathrm{M}}^3} + \frac{\boldsymbol{r}-\boldsymbol{r}_{\mathrm{M}}}{\left\|\boldsymbol{r}-\boldsymbol{r}_{\mathrm{M}}\right\|^3}\right) - \mu_{\mathrm{S}}\left(\frac{\boldsymbol{r}_{\mathrm{S}}}{r_{\mathrm{S}}^3} + \frac{\boldsymbol{r}-\boldsymbol{r}_{\mathrm{S}}}{\left\|\boldsymbol{r}-\boldsymbol{r}_{\mathrm{S}}\right\|^3}\right) + \frac{\boldsymbol{T}}{m_{\mathrm{sc}}} \tag{A.1}$$

$$0 \le T = \sqrt{T_x^2 + T_y^2 + T_z^2} \le 10\ \mathrm{N} \tag{A.2}$$

$$\dot{m}_{\mathrm{sc}} = -\frac{T}{g_{\mathrm{e}} I_{\mathrm{sp}}} \tag{A.3}$$

In equations (A.1~A.2), $\boldsymbol{r}$, $\boldsymbol{r}_{\mathrm{M}}$ and $\boldsymbol{r}_{\mathrm{S}}$ represent the position vectors of the probe, the moon and the sun relative to the Earth respectively; similarly, r, r_{M} and r_{S} represent the modules of position vectors of probe, the moon and the sun relative to the Earth respectively; μ_E, $\mu_{S,}$ and μ_M represents the gravitational constants of the earth, the sun and the moon respectively (see Appendix B), $\boldsymbol{T}$ is the thrust vector, g_{e} is the gravitational acceleration at sea level (see Appendix B), I_{sp} is the specific impulse (3000s), m_{sc} is the mass of probe.

2) Chemical thrust mode

Chemical thrust mode is approximated by several instantaneous velocity impulses.

Let the components on the three axis of ECI coordinate of an arbitrary velocity impulse be Δv_x, Δv_y, Δv_z and, let the moment before and after velocity impulse to be ($t_{\Delta v}^-$) and ($t_{\Delta v}^+$), the changes of position, velocity and mass of the probe satisfy the following equations:

$$x_{sc}(t_{\Delta v}^+) = x_{sc}(t_{\Delta v}^-), \qquad y_{sc}(t_{\Delta v}^+) = y_{sc}(t_{\Delta v}^-), \qquad z_{sc}(t_{\Delta v}^+) = z_{sc}(t_{\Delta v}^-) \tag{A.4}$$

$$\dot{x}_{sc}(t_{\Delta v}^+) = \dot{x}_{sc}(t_{\Delta v}^-) + \Delta v_x, \quad \dot{y}_{sc}(t^+) = \dot{y}_{sc}(t_{\Delta v}^-) + \Delta v_y, \quad \dot{z}_{sc}(t_{\Delta v}^+) = \dot{z}_{sc}(t_{\Delta v}^-) + \Delta v_z \tag{A.5}$$

$$m(t_{\Delta v}^+) = m(t_{\Delta v}^-)\exp\left(-\frac{\sqrt{\Delta v_x^2 + \Delta v_y^2 + \Delta v_z^2}}{g_e I_{sp}} \right) \tag{A.6}$$

$$\ddot{\boldsymbol{r}} = -\frac{\mu_E}{r^3}\boldsymbol{r} - \mu_M\left(\frac{\boldsymbol{r}_M}{r_M^3} + \frac{\boldsymbol{r} - \boldsymbol{r}_M}{\left\| \boldsymbol{r} - \boldsymbol{r}_M \right\|^3} \right) - \mu_S\left(\frac{\boldsymbol{r}_S}{r_S^3} + \frac{\boldsymbol{r} - \boldsymbol{r}_S}{\left\| \boldsymbol{r} - \boldsymbol{r}_S \right\|^3} \right) + \frac{T}{m_{sc}} \tag{A.7}$$

In equation (A.6), g_e is the gravitational acceleration at sea level (see Appendix B), I_{sp} is the specific impulse (400s).

When with no velocity impulse, the probe is influenced by the gravitation of the sun, the earth and the moon. The kinetic equations are as (A.7).

Appendix B. Definitions of Constants

Gravitational constant of the sun: μ_S=1.32712440018e11 km^3/s^2

Gravitational constant of the Earth: μ_E=398600 km^3/s^2

Gravitational constant of the moon: μ_M=4902.8 km^3/s^2

Gravitational acceleration of gravity on sea level: g_e =0.00980665 km^3/s^2

Earth's radius: 6378 km

One day (= 86400 seconds)

One year (= 365.25 days)

Corresponding MJD of January 1st 2021, 00:00 is: 59215

Corresponding MJD of December 31st 2030, 24:00 is: 62867

1AU=1.4959787066e8 km